# STATISTICAL METHODS FOR QUALITY CONTROL

**CONTENTS**

# DOW CHEMICAL U.S.A.*

FREEPORT, TEXAS

Dow Chemical U.S.A., Texas Operations, began in 1940 when The Dow Chemical Company purchased 800 acres of Texas land on the Gulf Coast to build a magnesium production facility. That original site has expanded to cover more than 5000 acres and is one of the largest petrochemical complexes in the world. Among the products from Texas Operations are magnesium, styrene, plastics, adhesives, solvent, glycol, and chlorine. Some products are made solely for use in other processes, but many end up as essential ingredients in products such as pharmaceuticals, toothpastes, dog food, water hoses, ice chests, milk cartons, garbage bags, shampoos, and furniture.

Dow's Texas Operations produces more than 30% of the world's magnesium, an extremely lightweight metal used in products ranging from tennis racquets to suitcases to "mag" wheels. The Magnesium Department was the first group in Texas Operations to train its technical people and managers in the use of statistical quality control. Some of the earliest successful applications of statistical quality control were in chemical processing.

*The authors are indebted to Clifford B. Wilson, Magnesium Technical Manager, The Dow Chemical Company, for providing this Statistics in Practice.

In one application involving the operation of a drier, samples of the output were taken at periodic intervals; the average value for each sample was computed and recorded on a chart called an $\bar{x}$ chart. Such a chart enabled Dow analysts to monitor trends in the output that might indicate the process was not operating correctly. In one instance, analysts began to observe values for the sample mean that were not indicative of a process operating within its design limits. On further examination of the control chart and the operation itself, the analysts found that the variation could be traced to problems involving one operator. The $\bar{x}$ chart recorded after that operator was retrained showed a significant improvement in the process quality.

Dow Chemical has achieved quality improvements everywhere statistical quality control has been used. Documented savings of several hundred thousand dollars per year have been realized, and new applications are continually being discovered.

In this chapter we will show how an $\bar{x}$ chart such as the one used by Dow Chemical can be developed. Such charts are a part of a statistical quality control known as statistical process control. We will also discuss methods of quality control for situations in which a decision to accept or reject a group of items is based on a sample.

The American Society for Quality (ASQ) defines **quality** as "the totality of features and characteristics of a product or service that bears on its ability to satisfy given needs." In other words, quality measures how well a product or service meets customer needs. Organizations recognize that to be competitive in today's global economy, they must strive for high levels of quality. As a result, an increased emphasis falls on methods for monitoring and maintaining quality.

**Quality assurance** refers to the entire system of policies, procedures, and guidelines established by an organization to achieve and maintain quality. Quality assurance consists of two principal functions: quality engineering and quality control. The objective of **quality engineering** is to include quality in the design of products and processes and to identify potential quality problems prior to production. **Quality control** consists of making a series of inspections and measurements to determine whether quality standards are being met. If quality standards are not being met, corrective and/or preventive action can be taken to achieve and maintain conformance. As we will show in this chapter, statistical techniques are extremely useful in quality control.

Traditional manufacturing approaches to quality control are being replaced by improved managerial tools and techniques. Competition with high-quality Japanese products has provided the impetus for this shift. Ironically, it was two U.S. consultants, Dr. W. Edwards Deming and Dr. Joseph Juran, who helped educate the Japanese in quality management.

*After World War II, Dr. W. Edwards Deming became a consultant to Japanese industry; he is credited with being the person who convinced top managers in Japan to use the methods of statistical quality control.*

Although quality is everybody's job, Deming stressed that quality improvements must be led by managers. He developed a list of 14 points that he believed are the key responsibilities of managers. For instance, Deming stated that managers must cease dependence on mass inspection; must end the practice of awarding business solely on the basis of price; must seek continual improvement in all production processes and services; must foster a team-oriented environment; and must eliminate numerical goals, slogans, and work standards that prescribe numerical quotas. Perhaps most important, managers must create a work environment in which a commitment to quality and productivity is maintained at all times.

In 1987, the U.S. Congress enacted Public Law 107, the Malcolm Baldrige National Quality Improvement Act. The Baldrige Award is given annually to U.S. firms that excel in quality. This award, along with the perspectives of individuals like Dr. Deming and Dr. Juran, has helped top managers recognize that improving service quality and product quality is the most critical challenge facing their companies. Winners of the Malcolm Baldrige Award include Motorola, IBM, Xerox, and FedEx. In this chapter we present two statistical methods used in quality control. The first method, **statistical process control,** uses graphical displays known as *control charts* to monitor a production process; the goal is to determine whether the process can be continued or whether it should be adjusted to achieve a desired quality level. The second method, **acceptance sampling,** is used in situations where a decision to accept or reject a group of items must be based on the quality found in a sample.

## 1 STATISTICAL PROCESS CONTROL

In this section we consider quality control procedures for a production process whereby goods are manufactured continuously. On the basis of sampling and inspection of production output, a decision will be made to either continue the production process or adjust it to bring the items or goods being produced up to acceptable quality standards.

*Continual improvement is one of the most important concepts of the total quality management movement. The most important use of a control chart is in improving the process.*

Despite high standards of quality in manufacturing and production operations, machine tools invariably wear out, vibrations throw machine settings out of adjustment, purchased materials contain defects, and human operators make mistakes. Any or all of these factors can result in poor quality output. Fortunately, procedures available to monitor production output help detect poor quality early, which allows for the adjustment and correction of the production process.

If the variation in the quality of the production output is due to **assignable causes** such as tools wearing out, incorrect machine settings, poor quality raw materials, or operator error, the process should be adjusted or corrected as soon as possible. Alternatively, if the variation is due to what are called **common causes**—that is, randomly occurring variations in materials, temperature, humidity, and so on, which the manufacturer cannot possibly control—the process does not need to be adjusted. The main objective of statistical process control is to determine whether variations in output are due to assignable causes or common causes.

Whenever assignable causes are detected, we conclude that the process is *out of control.* In that case, corrective action will be taken to bring the process back to an acceptable level of quality. However, if the variation in the output of a production process is due only to common causes, we conclude that the process is *in statistical control,* or simply *in control;* in such cases, no changes or adjustments are necessary.

*Process control procedures are based on hypothesis testing methodology. In essence, control charts provide an ongoing test of the hypothesis that the process is in control.*

The statistical procedures for process control are based on hypothesis testing methodology. The null hypothesis $H_0$ is formulated in terms of the production process being in control. The alternative hypothesis $H_a$ is formulated in terms of the production process being out of control. Table 1 shows that correct decisions to continue an in-control process and adjust an out-of-control process are possible. However, as with other hypothesis testing procedures, both a Type I error (adjusting an in-control process) and a Type II error (allowing an out-of-control process to continue) are also possible.

## Control Charts

*Control charts that are based on data that can be measured on a continuous scale are called variables control charts. The $\bar{x}$ chart is a variables control chart.*

A **control chart** provides a basis for deciding whether the variation in the output is due to common causes (in control) or assignable causes (out of control). Whenever an out-of-control situation is detected, adjustments and/or other corrective action will be taken to bring the process back into control.

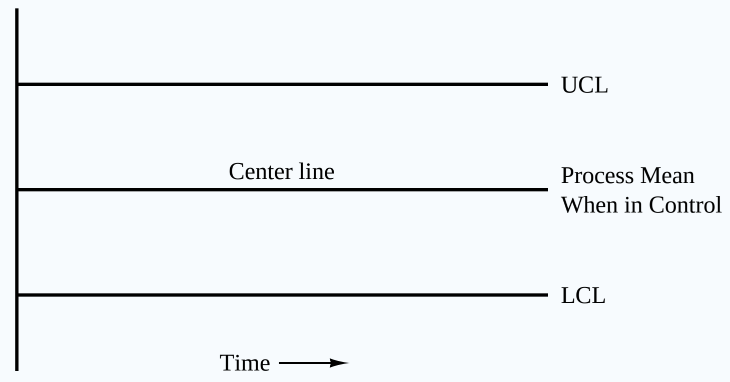
Control charts can be classified by the type of data they contain. An **$\bar{x}$ chart** is used if the quality of the output is measured in terms of a variable such as length, weight, temperature, and so on. In that case, the decision to continue or to adjust the production process will be based on the mean value found in a sample of the output. To introduce some of the concepts common to all control charts, let us consider some specific features of an $\bar{x}$ chart.

Figure 1 shows the general structure of an $\bar{x}$ chart. The center line of the chart corresponds to the mean of the process when the process is *in control.* The vertical line identi-

**TABLE 1** THE OUTCOMES OF STATISTICAL PROCESS CONTROL

| | | State of Production Process | |
|---|---|---|---|
| | | $H_0$ True<br>Process in Control | $H_0$ False<br>Process Out of Control |
| Decision | Continue Process | Correct decision | Type II error<br>(allowing an out-of-control process to continue) |
| | Adjust Process | Type I error<br>(adjusting an in-control process) | Correct decision |

FIGURE 1 $\bar{x}$ CHART STRUCTURE




fies the scale of measurement for the variable of interest. Each time a sample is taken from the production process, a value of the sample mean $\bar{x}$ is computed and a data point showing the value of $\bar{x}$ is plotted on the control chart.

The two lines labeled UCL and LCL are important in determining whether the process is in control or out of control. The lines are called the **upper control limit** and the **lower control limit,** respectively. They are chosen so that when the process is in control, there will be a high probability that the value of $\bar{x}$ will be between the two control limits. Values outside the control limits provide strong statistical evidence that the process is out of control and corrective action should be taken.

Over time, more and more data points will be added to the control chart. The order of the data points will be from left to right as the process is sampled. In essence, every time a point is plotted on the control chart, we are carrying out a hypothesis test to determine whether the process is in control.

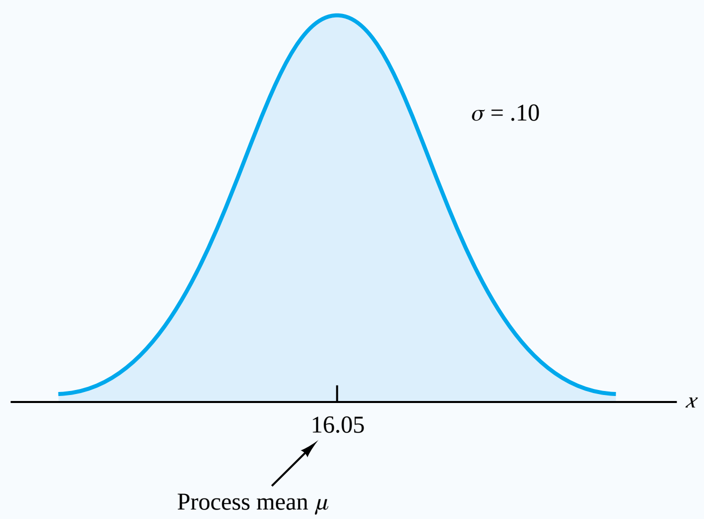
In addition to the $\bar{x}$ chart, other control charts can be used to monitor the range of the measurements in the sample (**R chart**), the proportion of defective items in the sample (***p* chart**), and the number of defective items in the sample (***np* chart**). In each case, the control chart has a LCL line, a center line, and a UCL line similar to the $\bar{x}$ chart in Figure 1. The major difference among the charts is what the vertical axis measures; for instance, in a *p* chart the vertical axis denotes the proportion of defective items in the sample instead of the sample mean. In the following discussion, we will illustrate the construction and use of the $\bar{x}$ chart, *R* chart, *p* chart, and *np* chart.

## $\bar{x}$ Chart: Process Mean and Standard Deviation Known

To illustrate the construction of an $\bar{x}$ chart, let us consider the situation at KJW Packaging. This company operates a production line where cartons of cereal are filled. Suppose KJW knows that when the process is operating correctly—and hence the system is in control—the mean filling weight is $\mu = 16.05$ ounces, and the process standard deviation is $\sigma = .10$ ounces. In addition, assume the filling weights are normally distributed. This distribution is shown in Figure 2.

The sampling distribution of $\bar{x}$ can be used to determine the expected variation in $\bar{x}$ values for a process that is in control. Let us first briefly review the properties of the sampling distribution of $\bar{x}$. First, recall that the expected value or mean of $\bar{x}$ is equal to $\mu$, the mean

**FIGURE 2** DISTRIBUTION OF CEREAL-CARTON FILLING WEIGHTS

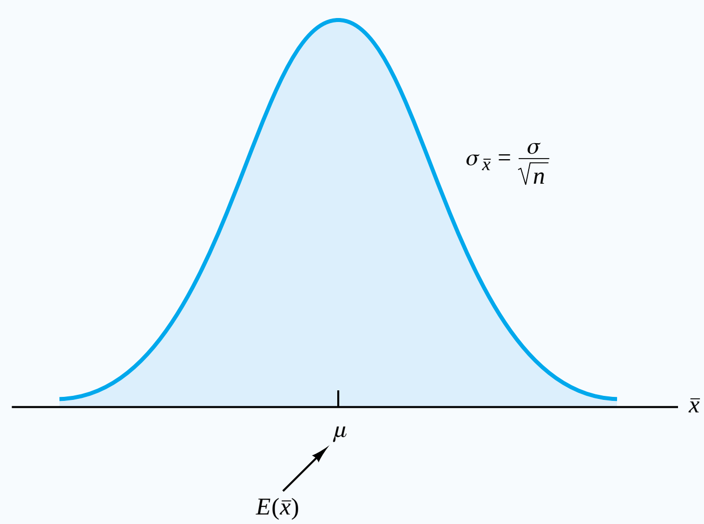



filling weight when the production line is in control. For samples of size $n$, the formula for the standard deviation of $\bar{x}$, called the standard error of the mean, is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \tag{1}$$

In addition, because the filling weights are normally distributed, the sampling distribution of $\bar{x}$ is normal for any sample size. Thus, the sampling distribution of $\bar{x}$ is a normal probability distribution with mean $\mu$ and standard deviation $\sigma_{\bar{x}}$. This distribution is shown in Figure 3.

**FIGURE 3** SAMPLING DISTRIBUTION OF $\bar{x}$



The sampling distribution of $\bar{x}$ is used to determine what values of $\bar{x}$ are reasonable if the process is in control. The general practice in quality control is to define as reasonable any value of $\bar{x}$ that is within 3 standard deviations above or below the mean value. Recall from the study of the normal probability distribution that approximately 99.7% of the values of a normally distributed random variable are within $\pm 3$ standard deviations of its mean value. Thus, if a value of $\bar{x}$ is within the interval $\mu - 3\sigma_{\bar{x}}$ to $\mu + 3\sigma_{\bar{x}}$, we will assume that the process is in control. In summary, then, the control limits for an $\bar{x}$ chart are as follows.

**Control Limits for an $\bar{x}$ Chart: Process Mean and Standard Deviation Known**

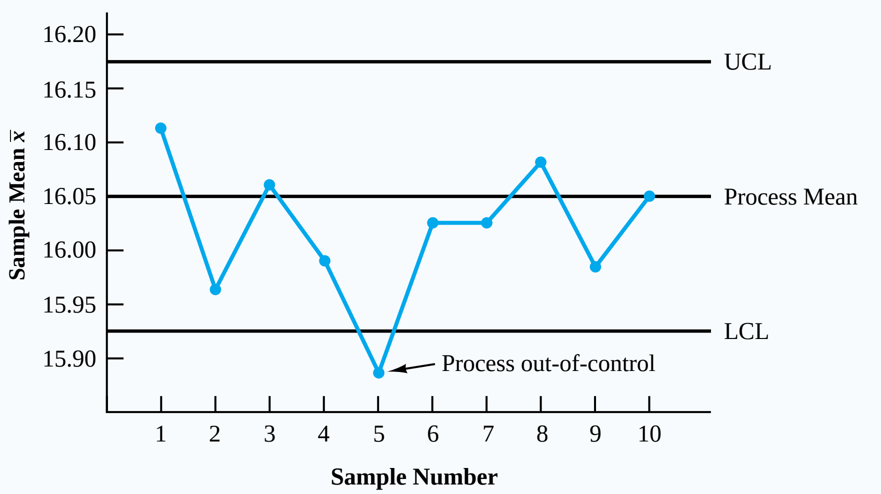
$$\text{UCL} = \mu + 3\sigma_{\bar{x}} \tag{2}$$

$$\text{LCL} = \mu - 3\sigma_{\bar{x}} \tag{3}$$

Reconsider the KJW Packaging example with the process distribution of filling weights shown in Figure 2 and the sampling distribution $\bar{x}$ of shown in Figure 3. Assume that a quality control inspector periodically samples six cartons and uses the sample mean filling weight to determine whether the process is in control or out of control. Using equation (1), we find that the standard error of the mean is $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .10/\sqrt{6} = .04$. Thus, with the process mean at 16.05, the control limits are UCL $= 16.05 + 3(.04) = 16.17$ and LCL $= 16.05 - 3(.04) = 15.93$. Figure 4 is the control chart with the results of 10 samples taken over a 10-hour period. For ease of reading, the sample numbers 1 through 10 are listed below the chart.

Note that the mean for the fifth sample in Figure 4 shows that the process is out of control. In other words, the fifth sample mean is below the LCL indicating that assignable causes of output variation are present and that underfilling is occurring. As a result, corrective action was taken at this point to bring the process back into control. The fact that the remaining points on the $\bar{x}$ chart are within the upper and lower control limits indicates that the corrective action was successful.

**FIGURE 4** $\bar{x}$ CHART FOR THE CEREAL-CARTON FILLING PROCESS



## $\bar{x}$ Chart: Process Mean and Standard Deviation Unknown

In the KJW Packaging example, we showed how an $\bar{x}$ chart can be developed when the mean and standard deviation of the process are known. In most situations, the process mean and standard deviation must be estimated by using samples that are selected from the process when it is in control. For instance, KJW might select a random sample of five boxes each morning and five boxes each afternoon for 10 days of in-control operation. For each subgroup, or sample, the mean and standard deviation of the sample are computed. The overall averages of both the sample means and the sample standard deviations can be used to construct control charts for both the process mean and the process standard deviation.

*It is important to maintain control over both the mean and the variability of a process.*

In practice, it is more common to monitor the variability of the process by using the range instead of the standard deviation because the range is easier to compute. The range data can be used to develop an estimate of $\sigma$, the process standard deviation; thus it can be used to construct upper and lower control limits for the $\bar{x}$ chart with little computational effort. To illustrate, let us consider the problem facing Jensen Computer Supplies, Inc.

Jensen Computer Supplies (JCS) manufactures 3.5-inch-diameter computer disks. Suppose random samples of five disks were selected during the first hour of operation, during the second hour of operation, and so on, until 20 samples were obtained. Table 2 provides the diameter of each disk sampled as well as the mean $\bar{x}_j$ and range $R_j$ for each of the samples. The process was believed to be in control during the sampling period.

The estimate of the process mean $\mu$ is given by the overall sample mean.

**Overall Sample Mean**

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k} \tag{4}$$

where

$\bar{x}_j$ = mean of the $j$th sample, $j = 1, 2, \ldots, k$
$k$ = number of samples

For the JCS data in Table 2, the overall sample mean is $\bar{\bar{x}} = 3.4995$. This value will be the center line for the $\bar{x}$ chart. The range of each sample, denoted $R_j$, is simply the difference between the largest and smallest values in each sample. The average range follows.

**Average Range**

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_k}{k} \tag{5}$$

where

$R_j$ = range of the $j$th sample, $j = 1, 2, \ldots, k$
$k$ = number of samples

For the JCS data in Table 2, the average range is $\bar{R} = .0253$.

**TABLE 2** DATA FOR THE JENSEN COMPUTER SUPPLIES PROBLEM

CDfile Jensen

| Sample Number | Observations | | | | | Sample Mean $\bar{x}_j$ | Sample Range $R_j$ |
|---|---|---|---|---|---|---|---|
| 1 | 3.5056 | 3.5086 | 3.5144 | 3.5009 | 3.5030 | 3.5065 | .0135 |
| 2 | 3.4882 | 3.5085 | 3.4884 | 3.5250 | 3.5031 | 3.5026 | .0368 |
| 3 | 3.4897 | 3.4898 | 3.4995 | 3.5130 | 3.4969 | 3.4978 | .0233 |
| 4 | 3.5153 | 3.5120 | 3.4989 | 3.4900 | 3.4837 | 3.5000 | .0316 |
| 5 | 3.5059 | 3.5113 | 3.5011 | 3.4773 | 3.4801 | 3.4951 | .0340 |
| 6 | 3.4977 | 3.4961 | 3.5050 | 3.5014 | 3.5060 | 3.5012 | .0099 |
| 7 | 3.4910 | 3.4913 | 3.4976 | 3.4831 | 3.5044 | 3.4935 | .0213 |
| 8 | 3.4991 | 3.4853 | 3.4830 | 3.5083 | 3.5094 | 3.4970 | .0264 |
| 9 | 3.5099 | 3.5162 | 3.5228 | 3.4958 | 3.5004 | 3.5090 | .0270 |
| 10 | 3.4880 | 3.5015 | 3.5094 | 3.5102 | 3.5146 | 3.5047 | .0266 |
| 11 | 3.4881 | 3.4887 | 3.5141 | 3.5175 | 3.4863 | 3.4989 | .0312 |
| 12 | 3.5043 | 3.4867 | 3.4946 | 3.5018 | 3.4784 | 3.4932 | .0259 |
| 13 | 3.5043 | 3.4769 | 3.4944 | 3.5014 | 3.4904 | 3.4935 | .0274 |
| 14 | 3.5004 | 3.5030 | 3.5082 | 3.5045 | 3.5234 | 3.5079 | .0230 |
| 15 | 3.4846 | 3.4938 | 3.5065 | 3.5089 | 3.5011 | 3.4990 | .0243 |
| 16 | 3.5145 | 3.4832 | 3.5188 | 3.4935 | 3.4989 | 3.5018 | .0356 |
| 17 | 3.5004 | 3.5042 | 3.4954 | 3.5020 | 3.4889 | 3.4982 | .0153 |
| 18 | 3.4959 | 3.4823 | 3.4964 | 3.5082 | 3.4871 | 3.4940 | .0259 |
| 19 | 3.4878 | 3.4864 | 3.4960 | 3.5070 | 3.4984 | 3.4951 | .0206 |
| 20 | 3.4969 | 3.5144 | 3.5053 | 3.4985 | 3.4885 | 3.5007 | .0259 |

In the preceding section we showed that the upper and lower control limits for the $\bar{x}$ chart are

$$\bar{x} \pm 3\frac{\sigma}{\sqrt{n}} \tag{6}$$

Hence, to construct the control limits for the $\bar{x}$ chart, we need to estimate $\sigma$, the standard deviation of the process. An estimate of $\sigma$ can be developed using the range data.

It can be shown that an estimator of the process standard deviation $\sigma$ is the average range divided by $d_2$, a constant that depends on the sample size $n$. That is,

$$\text{Estimator of } \sigma = \frac{\bar{R}}{d_2} \tag{7}$$

The American Society for Testing and Materials (ASTM) *Manual on Presentation of Data and Control Chart Analysis* provides values for $d_2$ as shown in Table 3. For instance, when $n = 5$, $d_2 = 2.326$, and the estimate of $\sigma$ is the average range divided by 2.326. If we substitute $\bar{R}/d_2$ for $\sigma$ in equation (6), we can write the control limits for the $\bar{x}$ chart as

$$\bar{\bar{x}} \pm 3\frac{\bar{R}/d_2}{\sqrt{n}} = \bar{\bar{x}} \pm \frac{3}{d_2\sqrt{n}}\bar{R} = \bar{\bar{x}} \pm A_2\bar{R} \tag{8}$$

Note that $A_2 = 3/(d_2\sqrt{n})$ is a constant that depends only on the sample size. Values for $A_2$ are provided in Table 3. For $n = 5$, $A_2 = .577$; thus, the control limits for the $\bar{x}$ chart are

$$3.4995 \pm (.577)(.0253) = 3.4995 \pm .0146$$

Hence, UCL = 3.514 and LCL = 3.485.

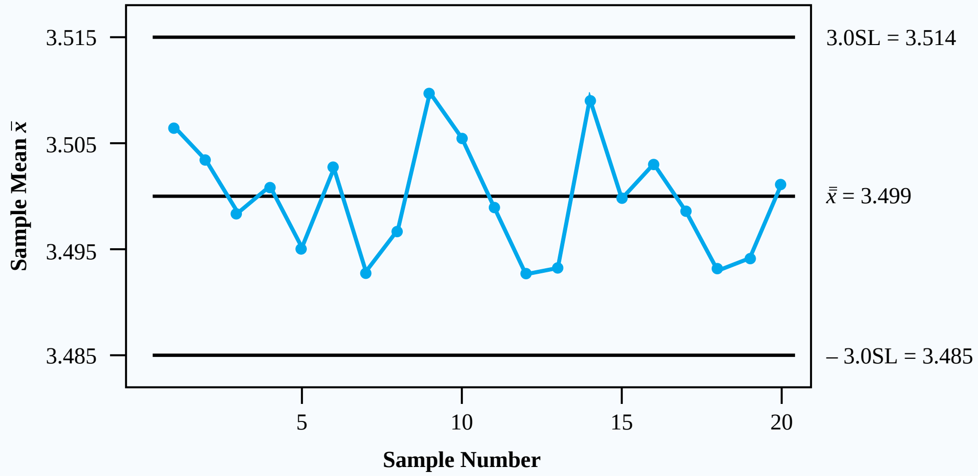
Figure 5 shows the $\bar{x}$ chart for the Jensen Computer Supplies problem. We used the data in Table 2 and Minitab's control chart routine to construct the chart. The center line is shown at the overall sample mean $\bar{\bar{x}} = 3.499$. The upper control limit (UCL) is 3.514. Minitab uses the notation 3.0SL to indicate the UCL is 3 standard deviations or 3 "sigma limits" (SL) above $\bar{\bar{x}}$. The lower control (LCL) is 3.485, which is −3.0SL or 3 "sigma limits" below $\bar{\bar{x}}$. The $\bar{x}$ chart shows the 20 sample means plotted over time. Because all 20 sample means are within the control limits, we confirm that the Jensen manufacturing process was in control during the sampling period. This chart can now be used to monitor the process mean on an ongoing basis.

**TABLE 3** FACTORS FOR $\bar{x}$ AND $R$ CONTROL CHARTS

| **Observations in Sample, *n*** | $d_2$ | $A_2$ | $d_3$ | $D_3$ | $D_4$ |
|---|---|---|---|---|---|
| 2 | 1.128 | 1.880 | 0.853 | 0 | 3.267 |
| 3 | 1.693 | 1.023 | 0.888 | 0 | 2.574 |
| 4 | 2.059 | 0.729 | 0.880 | 0 | 2.282 |
| 5 | 2.326 | 0.577 | 0.864 | 0 | 2.114 |
| 6 | 2.534 | 0.483 | 0.848 | 0 | 2.004 |
| 7 | 2.704 | 0.419 | 0.833 | 0.076 | 1.924 |
| 8 | 2.847 | 0.373 | 0.820 | 0.136 | 1.864 |
| 9 | 2.970 | 0.337 | 0.808 | 0.184 | 1.816 |
| 10 | 3.078 | 0.308 | 0.797 | 0.223 | 1.777 |
| 11 | 3.173 | 0.285 | 0.787 | 0.256 | 1.744 |
| 12 | 3.258 | 0.266 | 0.778 | 0.283 | 1.717 |
| 13 | 3.336 | 0.249 | 0.770 | 0.307 | 1.693 |
| 14 | 3.407 | 0.235 | 0.763 | 0.328 | 1.672 |
| 15 | 3.472 | 0.223 | 0.756 | 0.347 | 1.653 |
| 16 | 3.532 | 0.212 | 0.750 | 0.363 | 1.637 |
| 17 | 3.588 | 0.203 | 0.744 | 0.378 | 1.622 |
| 18 | 3.640 | 0.194 | 0.739 | 0.391 | 1.608 |
| 19 | 3.689 | 0.187 | 0.734 | 0.403 | 1.597 |
| 20 | 3.735 | 0.180 | 0.729 | 0.415 | 1.585 |
| 21 | 3.778 | 0.173 | 0.724 | 0.425 | 1.575 |
| 22 | 3.819 | 0.167 | 0.720 | 0.434 | 1.566 |
| 23 | 3.858 | 0.162 | 0.716 | 0.443 | 1.557 |
| 24 | 3.895 | 0.157 | 0.712 | 0.451 | 1.548 |
| 25 | 3.931 | 0.153 | 0.708 | 0.459 | 1.541 |

*Source:* Adapted from Table 27 of ASTM STP 15D, *ASTM Manual on Presentation of Data and Control Chart Analysis.* Copyright 1976 American Society of Testing and Materials, Philadelphia, PA. Reprinted with permission.

FIGURE 5 $\bar{x}$ CHART FOR THE JENSEN COMPUTER SUPPLIES PROBLEM




## *R* Chart

*To monitor process variability, a sample standard deviation control chart (s chart) can be constructed instead of an R chart. If the sample size is 10 or less, the R chart and the s chart provide similar results. If the sample size is greater than 10, the s chart is generally preferred.*

Let us now consider a range chart (*R* chart) that can be used to control the variability of a process. To develop the *R* chart, we need to think of the range of a sample as a random variable with its own mean and standard deviation. The average range $\bar{R}$ provides an estimate of the mean of this random variable. Moreover, it can be shown that an estimate of the standard deviation of the range is

$$\hat{\sigma}_R = d_3 \frac{\bar{R}}{d_2} \tag{9}$$

where $d_2$ and $d_3$ are constants that depend on the sample size; values of $d_2$ and $d_3$ are also provided in Table 3. Thus, the UCL for the *R* chart is given by

$$\bar{R} + 3\hat{\sigma}_R = \bar{R}\left(1 + 3\frac{d_3}{d_2}\right) \tag{10}$$

and the LCL is

$$\bar{R} - 3\hat{\sigma}_R = \bar{R}\left(1 - 3\frac{d_3}{d_2}\right) \tag{11}$$

If we let

$$D_4 = 1 + 3\frac{d_3}{d_2} \tag{12}$$

$$D_3 = 1 - 3\frac{d_3}{d_2} \tag{13}$$

we can write the control limits for the $R$ chart as

$$\text{UCL} = \bar{R}D_4 \tag{14}$$
$$\text{LCL} = \bar{R}D_3 \tag{15}$$

Values for $D_3$ and $D_4$ are also provided in Table 3. Note that for $n = 5$, $D_3 = 0$ and $D_4 = 2.114$. Thus, with $\bar{R} = .0253$, the control limits are

$$\text{UCL} = .0253(2.114) = .0534$$
$$\text{LCL} = .0253(0) = 0$$

*If the R chart indicates that the process is out of control, the $\bar{x}$ chart should not be interpreted until the R chart indicates the process variability is in control.*

Figure 6 shows the $R$ chart for the Jensen Computer Supplies problem. We used the data in Table 2 and Minitab's control chart routine to construct the chart. The center line is shown at the overall mean of the 20 sample ranges, $\bar{R} = .02527$. The UCL is .05344 or 3 sigma limits (3.0SL) above $\bar{R}$. The LCL is 0.0 or 3 sigma limits below $\bar{R}$. The $R$ chart shows the 20 sample ranges plotted over time. Because all 20 sample ranges are within the control limits, we confirm that the process was in control during the sampling period. This chart can now be used to monitor the process variability on an ongoing basis.

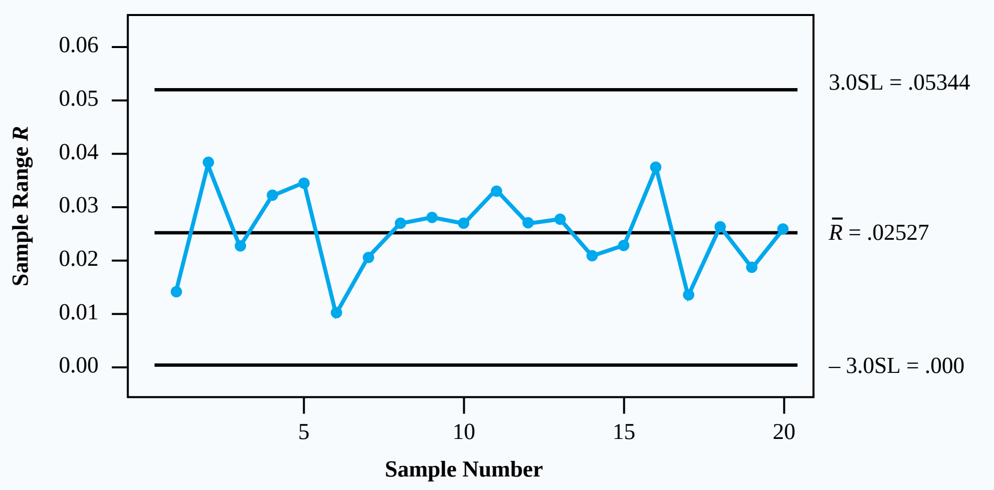
## *p* Chart

*Control charts that are based on data indicating the presence of a defect or a number of defects are called attributes control charts. A p chart is an attributes control chart.*

Let us consider the case in which the output quality is measured by either nondefective or defective items. The decision to continue or to adjust the production process will be based on $\bar{p}$, the proportion of defective items found in a sample. The control chart used for proportion-defective data is called a $p$ chart.

To illustrate the construction of a $p$ chart, consider the use of automated mail-sorting machines in a post office. These automated machines scan the zip codes on letters and divert each letter to its proper carrier route. Even when a machine is operating properly, some letters are diverted to incorrect routes. Assume that when a machine is operating correctly, or in a state of control, 3% of the letters are incorrectly diverted. Thus $p$, the proportion of letters incorrectly diverted when the process is in control, is .03.

**FIGURE 6** *R* CHART FOR THE JENSEN COMPUTER SUPPLIES PROBLEM



The sampling distribution of $\bar{p}$ can be used to determine the variation that can be expected in $\bar{p}$ values for a process that is in control. The expected value or mean of $\bar{p}$ is $p$, the proportion defective when the process is in control. With samples of size $n$, the formula for the standard deviation of $\bar{p}$, called the standard error of the proportion, is

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} \tag{16}$$

The sampling distribution of $\bar{p}$ can be approximated by a normal probability distribution whenever the sample size is large. With $\bar{p}$, the sample size can be considered large whenever the following two conditions are satisfied.

$$np \geq 5$$
$$n(1-p) \geq 5$$

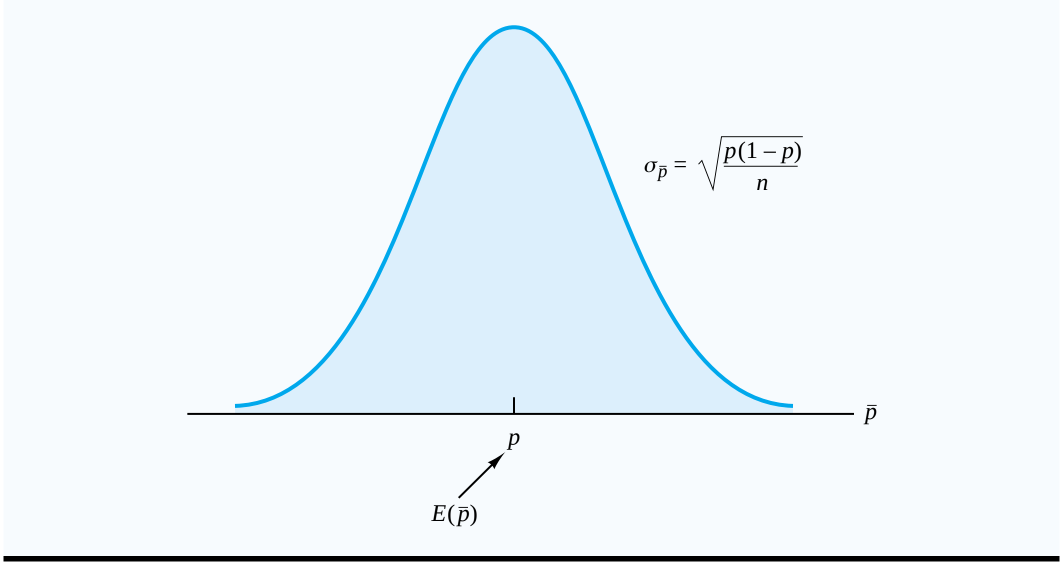
In summary, whenever the sample size is large, the sampling distribution of $\bar{p}$ can be approximated by a normal probability distribution with mean $p$ and standard deviation $\sigma_{\bar{p}}$. This distribution is shown in Figure 7.

To establish control limits for a $p$ chart, we follow the same procedure we used to establish control limits for an $\bar{x}$ chart. That is, the limits for the control chart are set at 3 standard deviations, or standard errors, above and below the proportion defective when the process is in control. Thus, we have the following control limits.

**Control Limits for a *p* Chart**

$$\text{UCL} = p + 3\sigma_{\bar{p}} \tag{17}$$
$$\text{LCL} = p - 3\sigma_{\bar{p}} \tag{18}$$

FIGURE 7 SAMPLING DISTRIBUTION OF $\bar{p}$



With $p = .03$ and samples of size $n = 200$, equation (16) shows that the standard error is

$$\sigma_{\bar{p}} = \sqrt{\frac{.03(1 - .03)}{200}} = .0121$$

Hence, the control limits are UCL $= .03 + 3(.0121) = .0663$, and LCL $= .03 - 3(.0121) = -.0063$. Because LCL is negative, LCL is set equal to zero in the control chart.

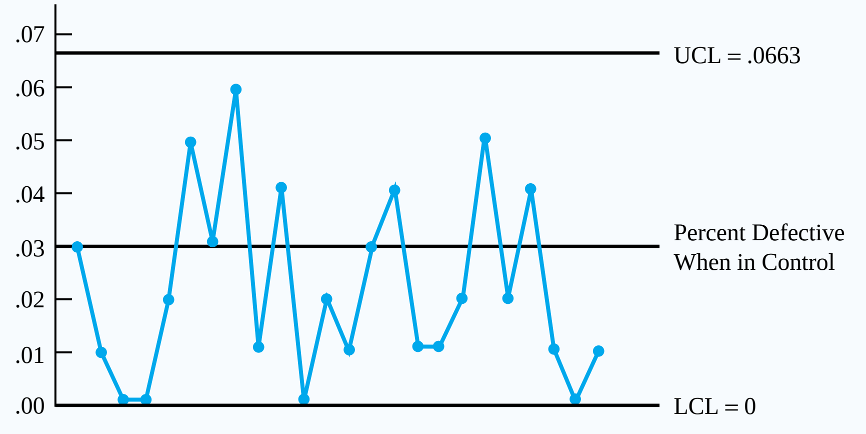
Figure 8 is the control chart for the mail-sorting process. The points plotted show the sample proportion defective found in samples of letters taken from the process. All points are within the control limits, providing no evidence to conclude that the sorting process is out of control. In fact, the $p$ chart indicates that the process is in control and should continue to operate.

If the proportion of defective items for a process that is in control is not known, that value is first estimated by using sample data. Suppose, for example, that $M$ different samples, each of size $n$, are selected from a process that is in control. The fraction or proportion of defective items in each sample is then determined. Treating all the data collected as one large sample, we can determine the average number of defective items for all the data; that value can then be used to provide an estimate of $p$, the proportion of defective items observed when the process is in control. Note that this estimate of $p$ also enables us to estimate the standard error of the proportion; upper and lower control limits can then be established.

## *np* Chart

An $np$ chart is a control chart developed for the number of defective items in a sample. In this case, $n$ is the sample size and $p$ is the probability of observing a defective item when the process is in control. Whenever the sample size is large, that is when $np \geq 5$ and $n(1 - p) \geq 5$, the distribution of the number of defective items observed in a sample of size $n$ can be approximated by a normal probability distribution with mean $np$ and standard deviation $\sqrt{np(1 - p)}$. Thus, for the mail-sorting example, with $n = 200$ and $p = .03$, the number of defective items observed in a sample of 200 letters can be approximated by a normal probability distribution with a mean of $200(.03) = 6$ and a standard deviation of $\sqrt{200(.03)(.97)} = 2.4125$.

**FIGURE 8** $p$ CHART FOR THE PROPORTION DEFECTIVE IN A MAIL-SORTING PROCESS



The control limits for an *np* chart are set at 3 standard deviations above and below the expected number of defective items observed when the process is in control. Thus, we have the following control limits.

**Control Limits for an *np* Chart**

$$\text{UCL} = np + 3\sqrt{np(1-p)} \quad (19)$$

$$\text{LCL} = np - 3\sqrt{np(1-p)} \quad (20)$$

For the mail-sorting process example, with $p = .03$ and $n = 200$, the control limits are UCL = 6 + 3(2.4125) = 13.2375, and LCL = 6 − 3(2.4125) = −1.2375. When LCL is negative, LCL is set equal to zero in the control chart. Hence, if the number of letters diverted to incorrect routes is greater than 13, the process is concluded to be out of control.

The information provided by an *np* chart is equivalent to the information provided by the *p* chart; the only difference is that the *np* chart is a plot of the number of defective items observed whereas the *p* chart is a plot of the proportion of defective items observed. Thus, if we were to conclude that a particular process is out of control on the basis of a *p* chart, the process would also be concluded to be out of control on the basis of an *np* chart.

## Interpretation of Control Charts

*Control charts are designed to identify when assignable causes of variation are present. Managers must then authorize action to eliminate the assignable cause and return the process to an in-control state.*

The location and pattern of points in a control chart enable us to determine, with a small probability of error, whether a process is in statistical control. A primary indication that a process may be out of control is a data point outside the control limits, such as the point corresponding to sample number 5 in Figure 4. Finding such a point is statistical evidence that the process is out of control; in such cases, corrective action should be taken as soon as possible.

In addition to points outside the control limits, certain patterns of the points within the control limits can be warning signals of quality control problems. For example, assume that all the data points are within the control limits but that a large number of points are on one side of the center line. This pattern may indicate that an equipment problem, a change in materials, or some other assignable cause of a shift in quality has occurred. Careful investigation of the production process should be undertaken to determine whether quality has changed.

*Even if all points are within the upper and lower control limits, a process may not be in control. Trends in the sample data points or unusually long runs above or below the center line may also indicate out-of-control conditions.*

Another pattern to watch for in control charts is a gradual shift, or trend, over time. For example, as tools wear out, the dimensions of machined parts will gradually deviate from their designed levels. Gradual changes in temperature or humidity, general equipment deterioration, dirt buildup, or operator fatigue may also result in a trend pattern in control charts. Six or seven points in a row that indicate either an increasing or decreasing trend should be cause for concern, even if the data points are all within the control limits. When such a pattern occurs, the process should be reviewed for possible changes or shifts in quality. Corrective action to bring the process back into control may be necessary.

### NOTES AND COMMENTS

1. Because the control limits for the $\bar{x}$ chart depend on the value of the average range, these limits will not have much meaning unless the process variability is in control. In practice, the *R* chart is usually constructed before the $\bar{x}$ chart; if the *R* chart indicates that the process variability is in control, then the $\bar{x}$ chart is constructed. Minitab's Xbar-R option provides the $\bar{x}$ chart and the

*R* chart simultaneously. The steps of this procedure are described in Appendix 1.

2. An *np* chart is used to monitor a process in terms of the number of defects. The Motorola Six Sigma Quality Level sets a goal of producing no more than 3.4 defects per million operations (*American Production and Inventory Control*, July 1991); this goal implies $p = .0000034$.

## EXERCISES

### Methods

**1.** A process that is in control has a mean of $\mu = 12.5$ and a standard deviation of $\sigma = .8$.

**a.** Construct an $\bar{x}$ chart if samples of size 4 are to be used.

**b.** Repeat part (a) for samples of size 8 and 16.

**c.** What happens to the limits of the control chart as the sample size is increased? Discuss why this is reasonable.

**2.** Twenty-five samples, each of size 5, were selected from a process that was in control. The sum of all the data collected was 677.5 pounds.

**a.** What is an estimate of the process mean (in terms of pounds per unit) when the process is in control?

**b.** Develop the control chart for this process if samples of size 5 be used. Assume that the process standard deviation is .5 when the process is in control, and that the mean of the process is the estimate developed in part (a).

**3.** Twenty-five samples of 100 items each were inspected when a process was considered to be operating satisfactorily. In the 25 samples, a total of 135 items were found to be defective.

**a.** What is an estimate of the proportion defective when the process is in control?

**b.** What is the standard error of the proportion if samples of size 100 will be used for statistical process control?

**c.** Compute the upper and lower control limits for the control chart.

**SELF**test

**4.** A process sampled 20 times with a sample of size 8 resulted in $\bar{\bar{x}} = 28.5$ and $\bar{R} = 1.6$. Compute the upper and lower control limits for the $\bar{x}$ and *R* charts for this process.

### Applications

**5.** Temperature is used to measure the output of a production process. When the process is in control, the mean of the process is $\mu = 128.5$ and the standard deviation is $\sigma = .4$.

**a.** Construct an $\bar{x}$ chart if samples of size 6 are to be used.

**b.** Is the process in control for a sample providing the following data?

128.8 128.2 129.1 128.7 128.4 129.2

**c.** Is the process in control for a sample providing the following data?

129.3 128.7 128.6 129.2 129.5 129.0

**6.** A quality control process monitors the weight per carton of laundry detergent. Control limits are set at UCL = 20.12 ounces and LCL = 19.90 ounces. Samples of size 5 are used for the sampling and inspection process. What are the process mean and process standard deviation for the manufacturing operation?

**7.** The Goodman Tire and Rubber Company periodically tests its tires for tread wear under simulated road conditions. To study and control the manufacturing process, 20 samples, each containing three radial tires, were chosen from different shifts over several days of operation, with the following results. Assuming that these data were collected when the manufacturing process was believed to be operating in control, develop the *R* and $\bar{x}$ charts.

CDfile
Tires

| Sample | Tread Wear* | | |
|---|---|---|---|
| 1 | 31 | 42 | 28 |
| 2 | 26 | 18 | 35 |
| 3 | 25 | 30 | 34 |
| 4 | 17 | 25 | 21 |
| 5 | 38 | 29 | 35 |
| 6 | 41 | 42 | 36 |
| 7 | 21 | 17 | 29 |
| 8 | 32 | 26 | 28 |
| 9 | 41 | 34 | 33 |
| 10 | 29 | 17 | 30 |
| 11 | 26 | 31 | 40 |
| 12 | 23 | 19 | 25 |
| 13 | 17 | 24 | 32 |
| 14 | 43 | 35 | 17 |
| 15 | 18 | 25 | 29 |
| 16 | 30 | 42 | 31 |
| 17 | 28 | 36 | 32 |
| 18 | 40 | 29 | 31 |
| 19 | 18 | 29 | 28 |
| 20 | 22 | 34 | 26 |

*Hundredths of an inch

**8.** Over several weeks of normal, or in-control, operation, 20 samples of 150 packages each of synthetic-gut tennis strings were tested for breaking strength. A total of 141 packages of the 3000 tested failed to conform to the manufacturer's specifications.

- **a.** What is an estimate of the process proportion defective when the system is in control?
- **b.** Compute the upper and lower control limits for a *p* chart.
- **c.** Using the results of part (b), what conclusion should be drawn about the process if tests on a new sample of 150 packages find 12 defective? Do there appear to be assignable causes in this situation?
- **d.** Compute the upper and lower control limits for an *np* chart.
- **e.** Answer part (c) using the results of part (d).
- **f.** Which control chart would be preferred in this situation? Explain.

**9.** An automotive industry supplier produces pistons for several models of automobiles. Twenty samples, each consisting of 200 pistons, were selected when the process was known to be operating correctly. The numbers of defective pistons found in the samples follow.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 8 | 10 | 6 | 4 | 5 | 7 | 8 | 12 | 8 | 15 |
| 14 | 10 | 10 | 7 | 5 | 8 | 6 | 10 | 4 | 8 |

- **a.** What is an estimate of the proportion defective for the piston manufacturing process when it is in control?
- **b.** Construct a *p* chart for the manufacturing process, assuming each sample has 200 pistons.
- **c.** With the results of part (b), what conclusion should be made if a sample of 200 has 20 defective pistons?
- **d.** Compute the upper and lower control limits for an *np* chart.
- **e.** Answer part (c) using the results of part (d).

## 2 ACCEPTANCE SAMPLING

In acceptance sampling, the items of interest can be incoming shipments of raw materials or purchased parts as well as finished goods from final assembly. Suppose we want to decide whether to accept or reject a group of items on the basis of specified quality characteristics. In quality control terminology, the group of items is a **lot,** and **acceptance sampling** is a statistical method that enables us to base the accept-reject decision on the inspection of a sample of items from the lot.

The general steps of acceptance sampling are shown in Figure 9. After a lot is received, a sample of items is selected for inspection. The results of the inspection are compared to specified quality characteristics. If the quality characteristics are satisfied, the lot is accepted and sent to production or shipped to customers. If the lot is rejected, managers must decide on its disposition. In some cases, the decision may be to keep the lot and remove the unacceptable or nonconforming items. In other cases, the lot may be returned to the supplier at the supplier's expense; the extra work and cost placed on the supplier can motivate the supplier to provide high-quality lots. Finally, if the rejected lot consists of finished goods, the goods must be scrapped or reworked to meet acceptable quality standards.

*Acceptance sampling has the following advantages over 100% inspection:*

1. *Usually less expensive*
2. *Less product damage due to less handling and testing*
3. *Fewer inspectors required*
4. *The only approach possible if destructive testing must be used*

The statistical procedure of acceptance sampling is based on hypothesis testing methodology presented. The null and alternative hypotheses are stated as follows.

$$H_0\text{: Good-quality lot}$$
$$H_a\text{: Poor-quality lot}$$

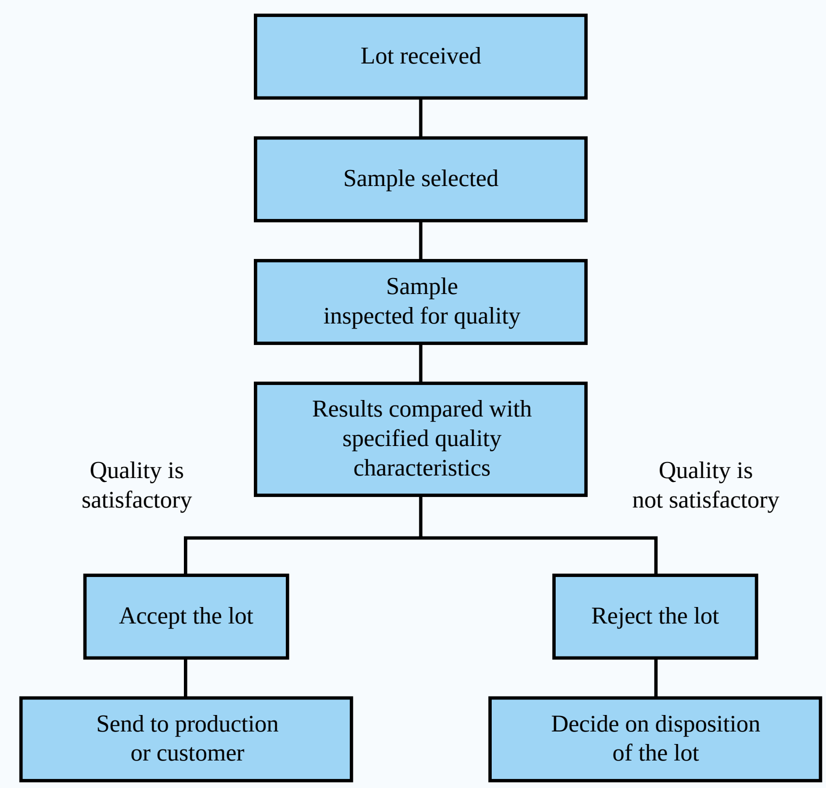
Table 4 shows the results of the hypothesis testing procedure. Note that correct decisions correspond to accepting a good-quality lot and rejecting a poor-quality lot. However, as with other hypothesis testing procedures, we need to be aware of the possibilities of making a Type I error (rejecting a good-quality lot) or a Type II error (accepting a poor-quality lot).

The probability of a Type I error creates a risk for the producer of the lot and is known as the **producer's risk.** For example, a producer's risk of .05 indicates a 5% chance that a good-quality lot will be erroneously rejected. The probability of a Type II error, on the other hand, creates a risk for the consumer of the lot and is known as the **consumer's risk.** For example, a consumer's risk of .10 means a 10% chance that a poor-quality lot will be erroneously accepted and thus used in production or shipped to the customer. Specific values for the producer's risk and the consumer's risk can be controlled by the person designing the acceptance sampling procedure. To illustrate how to assign risk values, let us consider the problem faced by KALI, Inc.

### KALI, Inc.: An Example of Acceptance Sampling

KALI, Inc., manufactures home appliances that are marketed under a variety of trade names. However, KALI does not manufacture every component used in its products. Several components are purchased directly from suppliers. For example, one of the components that KALI purchases for use in home air conditioners is an overload protector, a device that turns off the compressor if it overheats. The compressor can be seriously damaged if the overload protector does not function properly, and therefore KALI is concerned about the quality of the overload protectors. One way to ensure quality would be to test every component received; that approach is known as 100% inspection. However, to determine proper functioning of an overload protector, the device must be subjected to time-consuming and expensive tests, and KALI cannot justify testing every overload protector it receives.

**FIGURE 9** ACCEPTANCE SAMPLING PROCEDURE




Instead, KALI uses an acceptance sampling plan to monitor the quality of the overload protectors. The acceptance sampling plan requires that KALI's quality control inspectors select and test a sample of overload protectors from each shipment. If very few defective units are found in the sample, the lot is probably of good quality and should be accepted. However, if a large number of defective units are found in the sample, the lot is probably of poor quality and should be rejected.

**TABLE 4** OUTCOMES OF ACCEPTANCE SAMPLING

| | | State of the Lot | |
|---|---|---|---|
| | | **$H_0$ True Good-Quality Lot** | **$H_0$ False Poor-Quality Lot** |
| **Decision** | **Accept the Lot** | Correct decision | Type II error (accepting a poor-quality lot) |
| | **Reject the Lot** | Type I error (rejecting a good-quality lot) | Correct decision |

An acceptance sampling plan consists of a sample size $n$ and an acceptance criterion $c$. The **acceptance criterion** is the maximum number of defective items that can be found in the sample and still indicate an acceptable lot. For example, for the KALI problem let us assume that a sample of 15 items will be selected from each incoming shipment or lot. Furthermore, assume that the manager of quality control states that the lot can be accepted only if no defective items are found. In this case, the acceptance sampling plan established by the quality control manager is $n = 15$ and $c = 0$.

This acceptance sampling plan is easy for the quality control inspector to implement. The inspector simply selects a sample of 15 items, performs the tests, and reaches a conclusion based on the following decision rule.

- *Accept the lot* if zero defective items are found.
- *Reject the lot* if one or more defective items are found.

Before implementing this acceptance sampling plan, the quality control manager wants to evaluate the risks or errors possible under the plan. The plan will be implemented only if both the producer's risk (Type I error) and the consumer's risk (Type II error) are controlled at reasonable levels.

## Computing the Probability of Accepting a Lot

The key to analyzing both the producer's risk and the consumer's risk is a "What-if?" type of analysis. That is, we will assume that a lot has some known percentage of defective items and compute the probability of accepting the lot for a given sampling plan. By varying the assumed percentage of defective items, we can examine the effect of the sampling plan on both types of risks.

Let us begin by assuming that in a large shipment of overload protectors 5% of the overload protectors are defective. For a shipment or lot with 5% of the items defective, what is the probability that the $n = 15$, $c = 0$ sampling plan will lead us to accept the lot? Because each overload protector tested will be either defective or nondefective and because the lot size is large, the number of defective items in a sample of 15 has a *binomial probability distribution.* The binomial probability function follows.

**Binomial Probability Function for Acceptance Sampling**

$$f(x) = \frac{n!}{x!(n - x)!} p^x (1 - p)^{(n-x)} \tag{21}$$

where

$n$ = the sample size
$p$ = the proportion of defective items in the lot
$x$ = the number of defective items in the sample
$f(x)$ = the probability of $x$ defective items in the sample

For the KALI acceptance sampling plan, $n = 15$; thus, for a lot with 5% defective ($p = .05$), we have

$$f(x) = \frac{15!}{x!(15 - x)!} (.05)^x (1 - .05)^{(15-x)} \tag{22}$$

Using equation (22), $f(0)$ will provide the probability that zero overload protectors will be defective and the lot will be accepted. In using equation (22), recall that $0! = 1$. Thus, the probability computation for $f(0)$ is

$$f(0) = \frac{15!}{0!(15-0)!}(.05)^0(1-.05)^{(15-0)}$$
$$= \frac{15!}{0!(15)!}(.05)^0(.95)^{15} = (.95)^{15} = .4633$$

We now know that the $n = 15$, $c = 0$ sampling plan has a .4633 probability of accepting a lot with 5% defective items. Hence, there must be a corresponding $1 - .4633 = .5367$ probability of rejecting a lot with 5% defective items.

Tables of binomial probabilities can help reduce the computational effort in determining the probabilities of accepting lots. Selected binomial probabilities for $n = 15$ and $n = 20$ are listed in Table 5. Using this table, we can determine that if the lot contains 10% defective items, there is a .2059 probability that the $n = 15$, $c = 0$ sampling plan will lead us to accept the lot. The probability that the $n = 15$, $c = 0$ sampling plan will lead to the acceptance of lots with 1%, 2%, 3%, . . . defective items is summarized in Table 6.

Using the probabilities in Table 6, a graph of the probability of accepting the lot versus the percent defective in the lot can be drawn as shown in Figure 10. This graph, or curve, is called the **operating characteristic (OC) curve** for the $n = 15$, $c = 0$ acceptance sampling plan.

**TABLE 5** SELECTED BINOMIAL PROBABILITIES FOR SAMPLES OF SIZE 15 AND 20

| | | | | | | *p* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *n* | *x* | **.01** | **.02** | **.03** | **.04** | **.05** | **.10** | **.15** | **.20** | **.25** |
| 15 | 0 | .8601 | .7386 | .6333 | .5421 | .4633 | .2059 | .0874 | .0352 | .0134 |
| | 1 | .1303 | .2261 | .2938 | .3388 | .3658 | .3432 | .2312 | .1319 | .0668 |
| | 2 | .0092 | .0323 | .0636 | .0988 | .1348 | .2669 | .2856 | .2309 | .1559 |
| | 3 | .0004 | .0029 | .0085 | .0178 | .0307 | .1285 | .2184 | .2501 | .2252 |
| | 4 | .0000 | .0002 | .0008 | .0022 | .0049 | .0428 | .1156 | .1876 | .2252 |
| | 5 | .0000 | .0000 | .0001 | .0002 | .0006 | .0105 | .0449 | .1032 | .1651 |
| | 6 | .0000 | .0000 | .0000 | .0000 | .0000 | .0019 | .0132 | .0430 | .0917 |
| | 7 | .0000 | .0000 | .0000 | .0000 | .0000 | .0003 | .0030 | .0138 | .0393 |
| | 8 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0005 | .0035 | .0131 |
| | 9 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0007 | .0034 |
| | 10 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0007 |
| 20 | 0 | .8179 | .6676 | .5438 | .4420 | .3585 | .1216 | .0388 | .0115 | .0032 |
| | 1 | .1652 | .2725 | .3364 | .3683 | .3774 | .2702 | .1368 | .0576 | .0211 |
| | 2 | .0159 | .0528 | .0988 | .1458 | .1887 | .2852 | .2293 | .1369 | .0669 |
| | 3 | .0010 | .0065 | .0183 | .0364 | .0596 | .1901 | .2428 | .2054 | .1339 |
| | 4 | .0000 | .0006 | .0024 | .0065 | .0133 | .0898 | .1821 | .2182 | .1897 |
| | 5 | .0000 | .0000 | .0002 | .0009 | .0022 | .0319 | .1028 | .1746 | .2023 |
| | 6 | .0000 | .0000 | .0000 | .0001 | .0003 | .0089 | .0454 | .1091 | .1686 |
| | 7 | .0000 | .0000 | .0000 | .0000 | .0000 | .0020 | .0160 | .0545 | .1124 |
| | 8 | .0000 | .0000 | .0000 | .0000 | .0000 | .0004 | .0046 | .0222 | .0609 |
| | 9 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0011 | .0074 | .0271 |
| | 10 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0002 | .0020 | .0099 |
| | 11 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0005 | .0030 |
| | 12 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0000 | .0001 | .0008 |

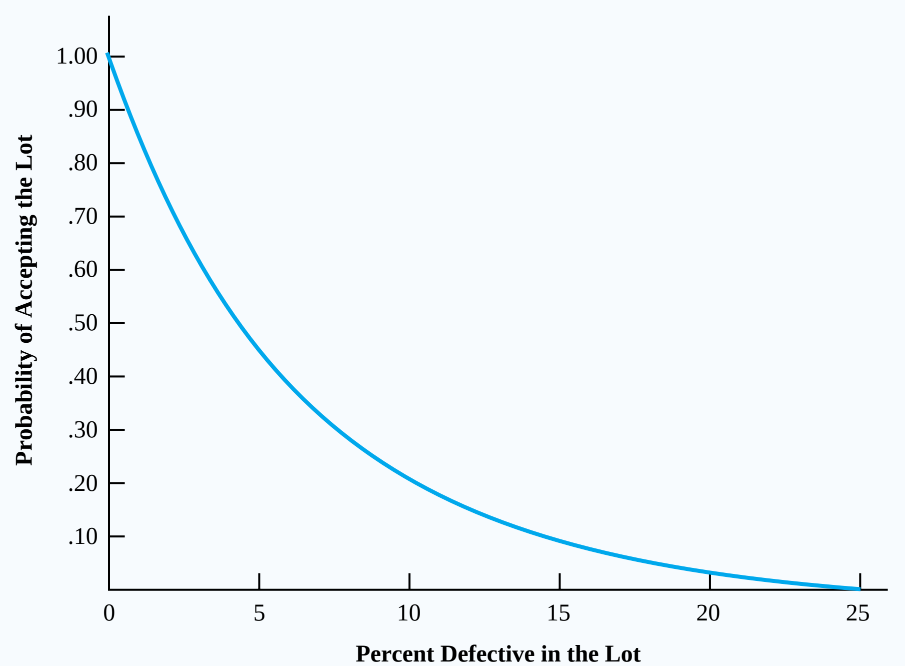
**TABLE 6** PROBABILITY OF ACCEPTING THE LOT FOR THE KALI PROBLEM WITH $n = 15$ and $c = 0$

| Percent Defective in the Lot | Probability of Accepting the Lot |
|---|---|
| 1 | .8601 |
| 2 | .7386 |
| 3 | .6333 |
| 4 | .5421 |
| 5 | .4633 |
| 10 | .2059 |
| 15 | .0874 |
| 20 | .0352 |
| 25 | .0134 |

Perhaps we should consider other sampling plans, ones with different sample sizes $n$ and/or different acceptance criteria $c$. First consider the case in which the sample size remains $n = 15$ but the acceptance criterion increases from $c = 0$ to $c = 1$. That is, we will now accept the lot if zero or one defective component is found in the sample. For a lot with 5% defective items ($p = .05$), Table 5 shows that with $n = 15$ and $p = .05$, $f(0) = .4633$ and $f(1) = .3658$. Thus, there is a $.4633 + .3658 = .8291$ probability that the $n = 15$, $c = 1$ plan will lead to the acceptance of a lot with 5% defective items.

Continuing these calculations we obtain Figure 11, which shows the operating characteristic curves for four alternative acceptance sampling plans for the KALI problem. Sam-

**FIGURE 10** OPERATING CHARACTERISTIC CURVE FOR THE $n = 15$, $c = 0$ ACCEPTANCE SAMPLING PLAN



ples of size 15 and 20 are considered. Note that regardless of the proportion defective in the lot, the $n = 15$, $c = 1$ sampling plan provides the highest probabilities of accepting the lot. The $n = 20$, $c = 0$ sampling plan provides the lowest probabilities of accepting the lot; however, that plan also provides the highest probabilities of rejecting the lot.

## Selecting an Acceptance Sampling Plan

Now that we know how to use the binomial probability distribution to compute the probability of accepting a lot with a given proportion defective, we are ready to select the values of $n$ and $c$ that determine the desired acceptance sampling plan for the application being studied. To do this, managers must specify two values for the fraction defective in the lot. One value, denoted $p_0$, will be used to control for the producer's risk, and the other value, denoted $p_1$, will be used to control for the consumer's risk.

In showing how to select the two values, we will use the following notation.

$\alpha$ = the producer's risk; the probability that a lot with $p_0$ defective will be rejected

$\beta$ = the consumer's risk; the probability that a lot with $p_1$ defective will be rejected

Suppose that for the KALI problem, the managers specify that $p_0 = .03$ and $p_1 = .15$. From the OC curve for $n = 15$, $c = 0$ in Figure 12, we see that $p_0 = .03$ provides a producer's risk of approximately $1 - .63 = .37$, and $p_1 = .15$ provides a consumer's risk of approximately .09. Thus, if the managers are willing to tolerate both a .37 probability of rejecting a lot with 3% defective items (producer's risk) and a .09 probability of accepting a lot with 15% defective items (consumer's risk), the $n = 15$, $c = 0$ acceptance sampling plan would be acceptable.

**FIGURE 11** OPERATING CHARACTERISTIC CURVES FOR FOUR ACCEPTANCE SAMPLING PLANS

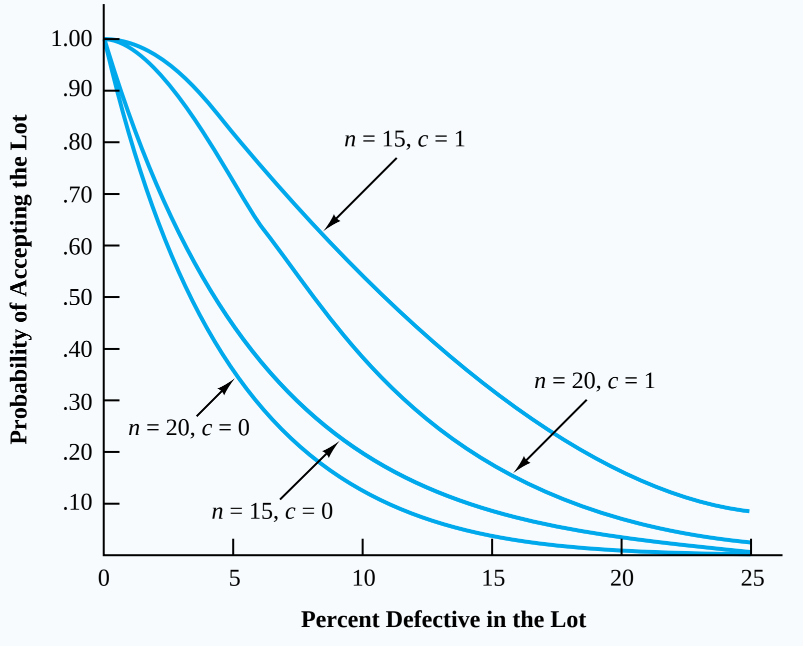
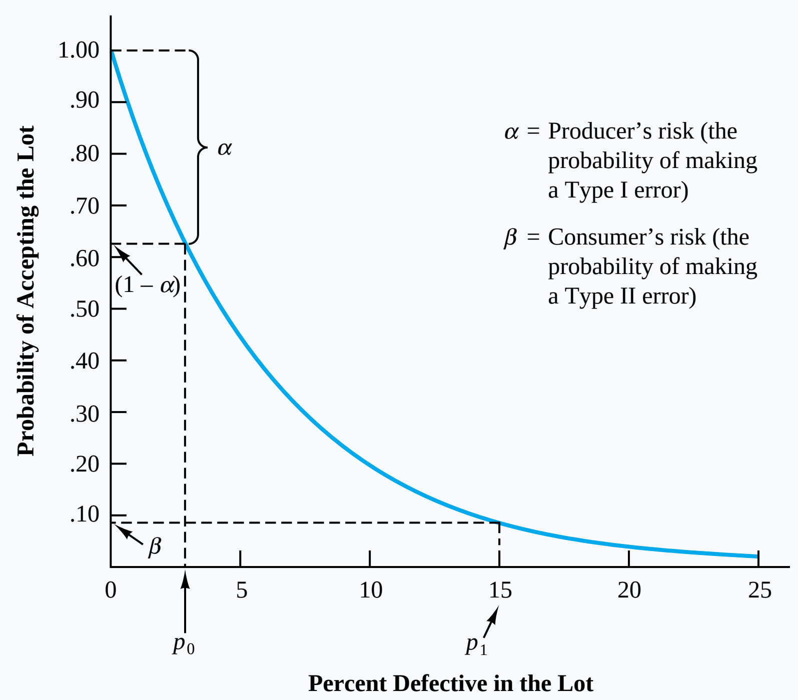


**FIGURE 12** OPERATING CHARACTERISTIC CURVE FOR $n = 15$, $c = 0$ with $p_0 = .03$ AND $p_1 = .15$




Suppose, however, that the managers request a producer's risk of $\alpha = .10$ and a consumer's risk of $\beta = .20$. We see that now the $n = 15$, $c = 0$ sampling plan has a better-than-desired consumer's risk but an unacceptably large producer's risk. The fact that $\alpha = .37$ indicates that 37% of the lots will be erroneously rejected when only 3% of the items in them are defective. The producer's risk is too high, and a different acceptance sampling plan should be considered.

Using $p_0 = .03$, $\alpha = .10$, $p_1 = .15$, and $\beta = .20$ in Figure 11 shows that the acceptance sampling plan with $n = 20$ and $c = 1$ comes closest to meeting both the producer's and the consumer's risk requirements. Exercise 13 at the end of this section will ask you to compute the producer's risk and the consumer's risk for the $n = 20$, $c = 1$ sampling plan.

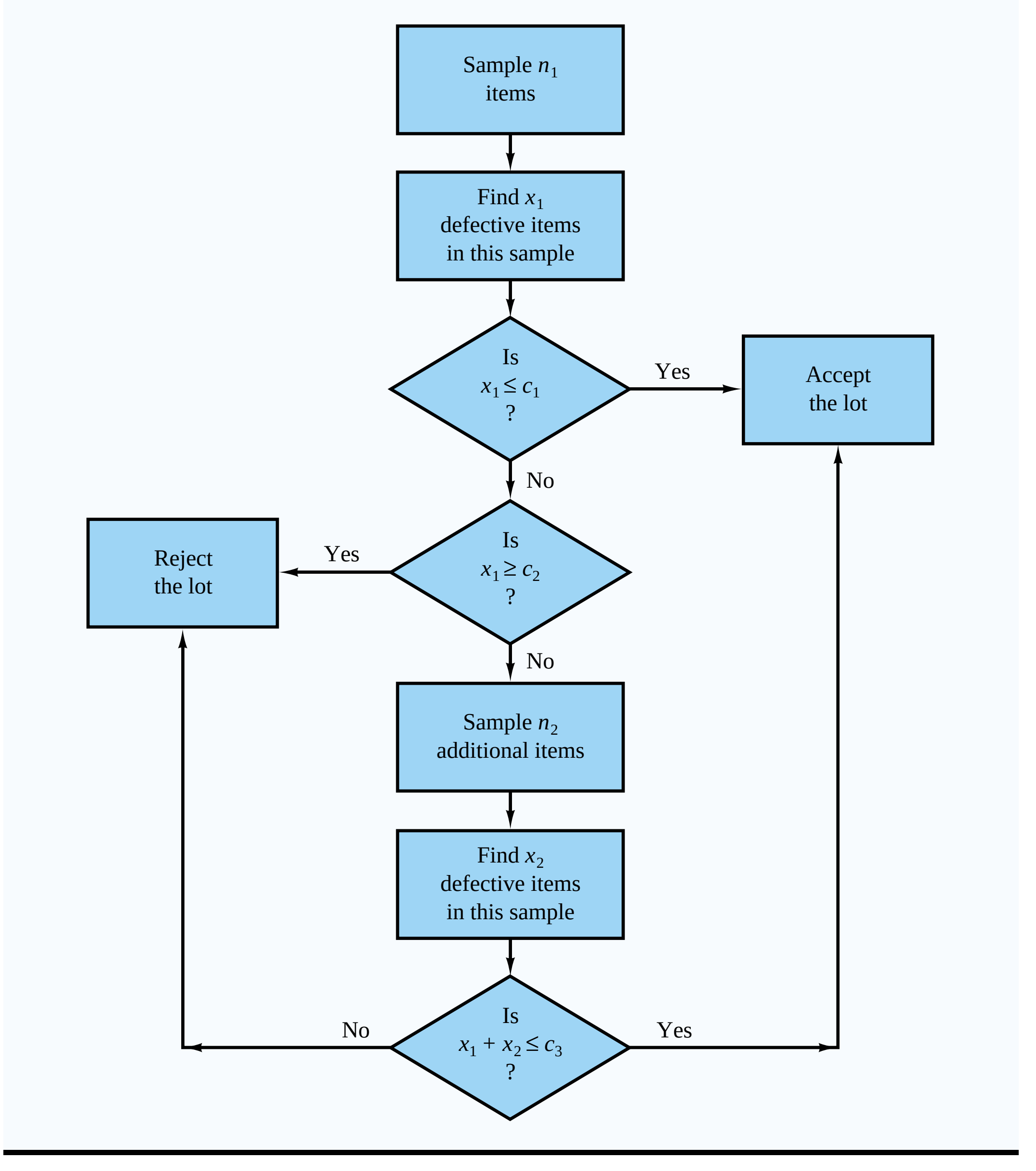
As shown in this section, several computations and several operating characteristic curves may need to be considered to determine the sampling plan with the desired producer's and consumer's risk. Fortunately, tables of sampling plans are published. For example, the American Military Standard Table, MIL-STD-105D, provides information helpful in designing acceptance sampling plans. More advanced texts on quality control describe the use of such tables. The advanced texts also discuss the role of sampling costs in determining the optimal sampling plan.

## Multiple Sampling Plans

The acceptance sampling procedure presented for the KALI problem is a *single-sample* plan. It is called a single-sample plan because only one sample or sampling stage is used. After the number of defective components in the sample is determined, a decision must be made to accept or reject the lot. An alternative to the single-sample plan is a **multiple sampling plan,** in which two or more stages of sampling are used. At each stage a decision is made among three possibilities: stop sampling and accept the lot, stop sampling and reject the lot, or continue sampling. Although more complex, multiple sampling plans often result in a smaller total sample size than single-sample plans with the same $\alpha$ and $\beta$ probabilities.

The logic of a two-stage, or double-sample, plan is shown in Figure 13. Initially a sample of $n_1$ items is selected. If the number of defective components $x_1$ is less than or equal to $c_1$, accept the lot. If $x_1$ is greater than or equal to $c_2$, reject the lot. If $x_1$ is between $c_1$ and $c_2$ ($c_1 < x_1 < c_2$), select a second sample of $n_2$ items. Determine the combined, or total,

**FIGURE 13** A TWO-STAGE ACCEPTANCE SAMPLING PLAN



number of defective components from the first sample ($x_1$) and the second sample ($x_2$). If $x_1 + x_2 \leq c_3$, accept the lot; otherwise reject the lot. The development of the double-sample plan is more difficult because the sample sizes $n_1$ and $n_2$ and the acceptance numbers $c_1$, $c_2$, and $c_3$ must meet both the producer's and consumer's risks desired.

## NOTES AND COMMENTS

1. The use of the binomial probability distribution for acceptance sampling is based on the assumption of large lots. If the lot size is small, the hypergeometric probability distribution is the appropriate distribution. Experts in the field of quality control indicate that the Poisson distribution provides a good approximation for acceptance sampling when the sample size is at least 16, the lot size is at least 10 times the sample size, and $p$ is less than .1. For larger sample sizes, the normal approximation to the binomial probability distribution can be used.
2. In the MIL-ST-105D sampling tables, $p_0$ is called the acceptable quality level (AQL). In some sampling tables, $p_1$ is called the lot tolerance percent defective (LTPD) or the rejectable quality level (RQL). Many of the published sampling plans also use quality indexes such as the indifference quality level (IQL) and the average outgoing quality limit (AOQL). More advanced texts provide a complete discussion of these other indexes.
3. In this section we provided an introduction to *attributes sampling plans*. In these plans each item sampled is classified as nondefective or defective. In *variables sampling plans,* a sample is taken and a measurement of the quality characteristic is taken. For example, for gold jewelry a measurement of quality may be the amount of gold it contains. A simple statistic such as the average amount of gold in the sample jewelry is computed and compared with an allowable value to determine whether to accept or reject the lot.

## EXERCISES

### Methods

SELFtest

**10.** For an acceptance sampling plan with $n = 25$ and $c = 0$, find the probability of accepting a lot with 2% defective items. What is the probability of accepting the lot if 6% of the items are defective?

**11.** Consider an acceptance sampling plan with $n = 20$ and $c = 0$. Compute the producer's risk for each of the following cases.

**a.** The lot contains 2% defective items.

**b.** The lot contains 6% defective items.

**12.** Repeat Exercise 11 for the acceptance sampling plan with $n = 20$ and $c = 1$. What happens to the producer's risk as the acceptance number $c$ is increased? Explain.

### Applications

**13.** Refer to the KALI problem presented in this section. The quality control manager requested a producer's risk of .10 when $p_0$ was .03 and a consumer's risk of .20 when $p_1$ was .15. Consider the acceptance sampling plan based on a sample size of 20 and an acceptance number of 1. Answer the following questions.

**a.** What is the producer's risk for the $n = 20$, $c = 1$ sampling plan?

**b.** What is the consumer's risk for the $n = 20$, $c = 1$ sampling plan?

**c.** Does the $n = 20$, $c = 1$ sampling plan satisfy the risks requested by the quality control manager? Discuss.

**14.** To inspect incoming shipments of raw materials, a manufacturer is considering samples of sizes 10, 15, and 20. Select a sampling plan that provides a producer's risk of $\alpha = .03$ when $p_0$ is .05 and a consumer's risk of $\beta = .12$ when $p_1$ is .30.

**15.** A domestic manufacturer of watches purchases quartz crystals from a Swiss firm. The crystals are shipped in lots of 1000. The acceptance sampling procedure uses 20 randomly selected crystals.

  **a.** Construct operating characteristic curves for acceptance numbers of 0, 1, and 2.

  **b.** If $p_0$ is .01 and $p_1 = .08$, what are the producer's and consumer's risks for each sampling plan in part (a)?

## SUMMARY

In this chapter we discussed how statistical methods can be used to assist in the control of quality. We first presented the $\bar{x}$, $R$, $p$, and $np$ control charts as graphical aids in monitoring process quality. Control limits are established for each chart; samples are selected periodically, and the data points are plotted on the control chart. Data points outside the control limits indicate that the process is out of control and that corrective action should be taken. Patterns of data points within the control limits can also indicate potential quality control problems and suggest that corrective action may be warranted.

We also considered the technique known as acceptance sampling. With this procedure, a sample is selected and inspected. The number of defective items in the sample provides the basis for accepting or rejecting the lot. The sample size and the acceptance criterion can be adjusted to control both the producer's risk (Type I error) and the consumer's risk (Type II error).

## GLOSSARY

**Quality control** A series of inspections and measurements used to determine whether quality standards are being met.

**Assignable causes** Variations in process outputs that are due to factors such as machine tools wearing out, incorrect machine settings, poor-quality raw materials, operator error, and so on. Corrective action should be taken when assignable causes of output variation are detected.

**Common causes** Normal or natural variations in process outputs that are due purely to chance. No corrective action is necessary when output variations are due to common causes.

**Control chart** A graphical tool used to help determine whether a process is in control or out of control.

**$\bar{x}$ chart** A control chart used to monitor the mean value of a variable such as a length, weight, temperature, and so on.

***R* chart** A control chart used to monitor the range of a variable.

***p* chart** A control chart used to monitor the proportion defective generated by a process.

***np* chart** A control chart used to monitor the number of defective items generated by a process.

**Lot** A group of items such as incoming shipments of raw materials or purchased parts as well as finished goods from final assembly.

**Acceptance sampling** A statistical procedure in which the number of defective items found in a sample is used to determine whether a lot should be accepted or rejected.

**Producer's risk** The risk of rejecting a good-quality lot; a Type I error.

**Consumer's risk** The risk of accepting a poor-quality lot; a Type II error.

**Acceptance criterion** The maximum number of defective items that can be found in the sample and still allow acceptance of the lot.

**Operating characteristic curve** A graph showing the probability of accepting the lot as a function of the percentage defective in the lot. This curve can be used to help determine whether a particular acceptance sampling plan meets both the producer's and the consumer's risk requirements.

**Multiple sampling plan** A form of acceptance sampling in which more than one sample or stage is used. On the basis of the number of defective items found in a sample, a decision will be made to accept the lot, reject the lot, or continue sampling.

## KEY FORMULAS

### Standard Error of the Mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \tag{1}$$

### Control Limits for an $\bar{x}$ Chart: Process Mean and Standard Deviation Known

$$\text{UCL} = \mu + 3\sigma_{\bar{x}} \tag{2}$$
$$\text{LCL} = \mu - 3\sigma_{\bar{x}} \tag{3}$$

### Overall Sample Mean

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_k}{k} \tag{4}$$

### Average Range

$$\bar{R} = \frac{R_1 + R_2 + \cdots + R_k}{k} \tag{5}$$

### Control Limits for an $\bar{x}$ Chart: Process Mean and Standard Deviation Unknown

$$\bar{\bar{x}} \pm A_2\bar{R} \tag{8}$$

### Control Limits for an *R* Chart

$$\text{UCL} = \bar{R}D_4 \tag{14}$$
$$\text{LCL} = \bar{R}D_3 \tag{15}$$

### Standard Error of the Proportion

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} \tag{16}$$

### Control Limits for a *p* Chart

$$\text{UCL} = p + 3\sigma_{\bar{p}} \tag{17}$$
$$\text{LCL} = p - 3\sigma_{\bar{p}} \tag{18}$$

### Control Limits for an *np* Chart

$$\text{UCL} = np + 3\sqrt{np(1-p)} \tag{19}$$
$$\text{LCL} = np - 3\sqrt{np(1-p)} \tag{20}$$

### Binomial Probability Function for Acceptance Sampling

$$f(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{(n-x)} \tag{21}$$

## SUPPLEMENTARY EXERCISES

**16.** Samples of size 5 provided the following 20 sample means for a production process that is believed to be in control.

| | | |
|---|---|---|
| 95.72 | 95.24 | 95.18 |
| 95.44 | 95.46 | 95.32 |
| 95.40 | 95.44 | 95.08 |
| 95.50 | 95.80 | 95.22 |
| 95.56 | 95.22 | 95.04 |
| 95.72 | 94.82 | 95.46 |
| 95.60 | 95.78 | |

**a.** Based on these data, what is an estimate of the mean when the process is in control?
**b.** Assuming that the process standard deviation is $\sigma = .50$, develop a control chart for this production process. Assume that the mean of the process is the estimate developed in part (a).
**c.** Are any of the 20 sample means outside the control limits?

**17.** Product filling weights are normally distributed with a mean of 350 grams and a standard deviation of 15 grams.
**a.** Develop the control limits for samples of size 10, 20, and 30.
**b.** What happens to the control limits as the sample size is increased?
**c.** What happens when a Type I error is made?
**d.** What happens when a Type II error is made?
**e.** What is the probability of a Type I error for samples of size 10, 20, and 30?
**f.** What is the advantage of increasing the sample size for control chart purposes? What error probability is reduced as the sample size is increased?

**18.** Twenty-five samples of size 5 resulted in in $\bar{\bar{x}} = 5.42$ and $\bar{R} = 2.0$. Compute control limits for the $\bar{x}$ and $R$ charts, and estimate the standard deviation of the process.

**19.** Twenty samples of size 5 were selected from a manufacturing process at Kensport Chemical Company. The following data show the mean temperature and range in degrees centigrade for each of the twenty samples. The process was believed to be in control when the samples were selected. The company is interested in using control charts to monitor the temperature of its manufacturing process. Construct the $\bar{x}$ chart and $R$ chart. Does it appear that the process was in control when the samples were selected?

| Sample | $\bar{x}$ | $R$ | Sample | $\bar{x}$ | $R$ |
|---|---|---|---|---|---|
| 1 | 95.72 | 1.0 | 11 | 95.80 | .6 |
| 2 | 95.24 | .9 | 12 | 95.22 | .2 |
| 3 | 95.18 | .8 | 13 | 95.56 | 1.3 |
| 4 | 95.44 | .4 | 14 | 95.22 | .5 |
| 5 | 95.46 | .5 | 15 | 95.04 | .8 |
| 6 | 95.32 | 1.1 | 16 | 95.72 | 1.1 |
| 7 | 95.40 | .9 | 17 | 94.82 | .6 |
| 8 | 95.44 | .3 | 18 | 95.46 | .5 |
| 9 | 95.08 | .2 | 19 | 95.60 | .4 |
| 10 | 95.50 | .6 | 20 | 95.74 | .6 |

**20.** The following were collected for the Master Blend Coffee production process. The data show the filling weights based on samples of 3-pound cans of coffee. Use these data to construct the $\bar{x}$ and $R$ chart. Does it appear that the process was in control when the samples were selected?

CDfile
Coffee

| | Observations | | | | |
|---|---|---|---|---|---|
| Sample | 1 | 2 | 3 | 4 | 5 |
| 1 | 3.05 | 3.08 | 3.07 | 3.11 | 3.11 |
| 2 | 3.13 | 3.07 | 3.05 | 3.10 | 3.10 |
| 3 | 3.06 | 3.04 | 3.12 | 3.11 | 3.10 |
| 4 | 3.09 | 3.08 | 3.09 | 3.09 | 3.07 |
| 5 | 3.10 | 3.06 | 3.06 | 3.07 | 3.08 |
| 6 | 3.08 | 3.10 | 3.13 | 3.03 | 3.06 |
| 7 | 3.06 | 3.06 | 3.08 | 3.10 | 3.08 |
| 8 | 3.11 | 3.08 | 3.07 | 3.07 | 3.07 |
| 9 | 3.09 | 3.09 | 3.08 | 3.07 | 3.09 |
| 10 | 3.06 | 3.11 | 3.07 | 3.09 | 3.07 |

**21.** Consider the following situations. Comment on whether the situation might cause concern about the process.

**a.** A $p$ chart has LCL = 0 and UCL = .068. When the process is in control, the proportion defective is .033. Plot the following seven sample results: .035, .062, .055, .049, .058, .066, and .055. Discuss.

**b.** An $\bar{x}$ chart has LCL = 22.2 and UCL = 24.5. The mean is $\mu$ = 23.35 when the process is in control. Plot the following seven sample results: 22.4, 22.6, 22.65, 23.2, 23.4, 23.85, and 24.1. Discuss.

**22.** Managers of 1200 different retail outlets make twice-a-month restocking orders from a central warehouse. Past experience shows that 4% of the orders contain one or more errors such as wrong item shipped, wrong quantity shipped, and item requested but not shipped. Random samples of 200 orders are selected monthly and checked for accuracy.

**a.** Construct a control chart for this situation.

**b.** Six months of data show the following numbers of orders with one or more errors: 10, 15, 6, 13, 8, and 17. Plot the data on the control chart. What does your plot indicate about the order process?

**23.** An $n = 10$, $c = 2$ acceptance sampling plan is being considered; assume that $p_0 = .05$ and $p_1 = .20$.

**a.** Compute both the producer's and the consumer's risk for this acceptance sampling plan.

**b.** Would either the producer, the consumer, or both be unhappy with the proposed sampling plan?

**c.** What change in the sampling plan, if any, would you recommend?

**24.** An acceptance sampling plan with $n = 15$ and $c = 1$ has been designed with a producer's risk of .075.

**a.** Was the value of $p_0$ .01, .02, .03, .04, or .05? What does this value mean?

**b.** What is the consumer's risk associated with this plan if $p_1$ is .25?

**25.** A manufacturer produces lots of a canned food product. Let $p$ denote the proportion of the lots that do not meet the product quality specifications. An $n = 25$, $c = 0$ acceptance sampling plan will be used.

**a.** Compute points on the operating characteristic curve when $p = .01, .03, .10$, and .20.

**b.** Plot the operating characteristic curve.

**c.** What is the probability that the acceptance sampling plan will reject a lot with .01 defective?

## Appendix 1 CONTROL CHARTS WITH MINITAB

**CDfile** Jensen

In this appendix we describe the steps required to generate Minitab control charts using the Jensen sample data shown in Table 2. The sample number appears in column C1. The first observation is in column C2, the second observation is in column C3, and so on. The following steps describe how to use Minitab to produce both the $\bar{x}$ chart and $R$ chart simultaneously.

Step 1. Select the **Stat** pull-down menu
Step 2. Choose **Control Charts**
Step 3. Choose **Xbar-R**
Step 4. When the Xbar-R Chart dialog box appears:
Select **Subgroups across rows of**
Enter C2-C6 in the Subgroups across rows of box
Select **Tests**
Step 5. When the Tests dialog box appears:
Choose **One point more than 3 sigmas from center line***
Click **OK**
Step 6. When the Xbar-R Chart dialog box appears
Click **OK**

The $\bar{x}$ chart and the $R$ chart will be shown together on the Minitab output. The choices available under step 3 of the preceding Minitab procedure provide access to a variety of control chart options. For example, the $\bar{x}$ and the $R$ chart can be selected separately. Additional options include the $p$ chart, the $np$ chart, and others.

*Minitab provides several additional tests for detecting special causes of variation and out-of-control conditions. The user may select several of these tests simultaneously.

# ANSWERS TO EVEN-NUMBERED EXERCISES

**2.** **a.** 5.42
**b.** UCL = 6.09, LCL = 4.75

**4.**

| | *R* Chart | $\bar{x}$ Chart |
|---|---|---|
| UCL | 2.98 | 29.10 |
| LCL | .22 | 27.90 |

**6.** 20.01, .082

**8.** **a.** .0470
**b.** UCL = .0989, LCL = 2.0049 (use LCL = 0)
**c.** $\bar{p} = .08$; in control
**d.** UCL = 14.826, LCL = −0.726
Process is out of control if more than 14 defective
**e.** In control with 12 defective
**f.** *np* chart

**10.** $p = .02$; $f(0) = .6035$
$p = .06$; $f(0) = .2129$

**12.** $p_0 = .02$; producer's risk = .0599
$p_0 = .06$; producer's risk = .3396
Producer's risk decreases as the acceptance number $c$ is increased

**14.** $n = 20$, $c = 3$

**16.** **a.** 95.4
**b.** UCL = 96.07, LCL = 94.73
**c.** No

**18.**

| | *R* Chart | $\bar{x}$ Chart |
|---|---|---|
| UCL | 4.23 | 6.57 |
| LCL | 0 | 4.27 |

Estimate of standard deviation = .86

**20.**

| | *R* Chart | $\bar{x}$ Chart |
|---|---|---|
| UCL | .1121 | 3.112 |
| LCL | 0 | 3.051 |

Because all data points are within the control limits for both charts, it does appear that the process was in control when the samples were selected.

**22.** **a.** UCL = .0817, LCL = −.0017 (use LCL = 0)

**24.** **a.** .03
**b.** $\beta = .0802$

# SOLUTIONS TO SELF-TEST EXERCISES

**4.** *R chart:*
UCL = $\bar{R}D_4 = 1.6(1.864) = 2.98$
LCL = $\bar{R}D_3 = 1.6(.136) = .22$
$\bar{x}$ chart:
UCL = $\bar{\bar{x}} + A_2\bar{R} = 28.5 + .373(1.6) = 29.10$
LCL = $\bar{\bar{x}} - A_2\bar{R} = 28.5 - .373(1.6) = 27.90$

**10.** $$f(0) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

When $p = .02$, the probability of accepting the lot is

$$f(0) = \frac{25!}{0!(25-0)!}(.02)^0(1-.02)^{25} = .6035$$

When $p = .06$, the probability of accepting the lot is

$$f(0) = \frac{25!}{0!(25-0)!}(.06)^0(1-.06)^{25} = .2129$$